COT2104 Answers Hwk 2 (parts 1 and 2)

1. a) f is not a function because there is no pair with the form (3, _)
   b) f is not a function because there is two pairs with the form (1, _)

c) f is a function

2. A : The set of students registered at U of C.
   B: The set of professors at U of C
   C: The set of courses currently being offered at U of C.

   a) {(a, b) | a's first class each week is in c}
   Assuming that each student currently registered at U of C is taking at least one course, then is a function.
   b) {(a, b) | a has a class in C Saturday evening}
   Assuming students have no classes on Saturday evening since the set would contain no pair (a, _), then it is not a function.
   Note: If there is a class on Saturday evening, it could be considered a function.

3. The inverse $f^{-1} = \{ (2, 1), (4,2), (3,3), (1,4)\}$
4. $f: R \rightarrow R$ given by $f(x) = x^3 - 2$

the inverse is

$f(x) = y$
$y = x^3 - 2$
$x = y^3 - 2$ change x and y
$y = (x + 2)^{1/3}$ solve for y
$y = f^{-1}(x)$

$f^{-1}: R \rightarrow R$ is defined by $f^{-1}(x) = (x + 2)^{1/3}$

5. Show that $f: A \rightarrow R$ is one-to-one. Find the range and its inverse

Suppose $f(x_1) = f(x_2)$ then $5 - 1 / (1 + x_1) = 5 - 1 / (1 + x_2)$
So $1 / (1 + x_1) = 1 / (1 + x_2)$
And $1 + x_1 = 1 + x_2$
$x_1 = x_2$. Thus f is one-to-one

$y \in$ rng $\leftrightarrow y = f(x)$ for some $x \in A$
$\leftrightarrow$ There is an $x \in A$ such that $y = 5 - 1 / (1 + x)$
$\leftrightarrow$ an $x \in A$ such that $y - 5 = - 1 / (1 + x)$
$\leftrightarrow$ an $x \in A$ such that $(y - 5) (1 + x) = - 1$
$\leftrightarrow y \neq 5$
Thus rng $f = B = \{y \in R \mid y \neq 5\}$ and f has an inverse $B \rightarrow A$

To find $f^{-1}(x)$, let $y = f^{-1}(x)$, $x \in B$
$f(x) = y$
$x = 5 - 1 / (y + 1)$

$$x - 5 = -1 / (y + 1)$$
$$(x - 5)(y + 1) = -1$$
$$y+1 = -1 / (x - 5) \qquad \text{since } x \neq 5$$
$$f^{-1}(x) = -1 - 1 / (x - 5)$$

6) Find a one-to-one correspondence between each of the following pairs of sets.

a) {x, y, {a, b, c}} and {14, -3, t}

$x \leftrightarrow 14$, $y \leftrightarrow -3$, $\{a, b, c\} \leftrightarrow t$

c) 2Z and 17Z

f: 2z $\rightarrow$ 17Z defined by f(2k) = 7k for $2k \in 2Z$

7)
a) $f^{-1} = \{(2, 1), (3, 2), (4, 3), (1, 4)\}$

b) $g \circ f$

$g \circ f\ f(1) = g(f(1)) = 1$
$g \circ f\ f(2) = g(f(2)) = 4$
$g \circ f\ f(3) = g(f(3)) = 2$
$g \circ f\ f(3) = g(f(4)) = 3$
$g \circ f = \{ (1,1), (2,4), (3,2), (4,3)\}$

c) g is not one-to-one because g(2) = g(5) , but $2 \neq 5$

8.
a) 9833 is prime
b) 150 is composite

9.
a) a = -5286; b = 19, b > 0 ← floor
q = $\lfloor -5286/19 \rfloor$ = -279, r = 15

b) a = 5286; b = -19, b > 0 ← ceiling
q = $\lceil 5286/-19 \rceil$ = -278, r = 4

10.

	2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
a) 57483_{10} =	1	1	1	0	0	0	0	0	1	0	0	0	1	0	1	1_2

	8^5	8^4	8^3	8^2	8^1	8^0
=	1	6	0	8	1	3_8

16^3 16^2 16^1 16^0

$= \; E \quad 0 \quad 8 \quad B_{16}$

b) $185{,}178_{10} = \begin{array}{cccccccccccccccccc} 2^{17} & 2^{16} & 2^{15} & 2^{14} & 2^{13} & 2^{12} & 2^{11} & 2^{10} & 2^{9} & 2^{8} & 2^{7} & 2^{6} & 2^{5} & 2^{4} & 2^{3} & 2^{2} & 2^{1} & 2^{0} \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0_2 \end{array}$

$= \begin{array}{cccccc} 8^5 & 8^4 & 8^3 & 8^2 & 8^1 & 8^0 \\ 5 & 5 & 1 & 5 & 3 & 2_8 \end{array}$

$= \begin{array}{ccccc} 16^4 & 16^3 & 16^2 & 16^1 & 16^0 \\ 2 & D & 3 & 5 & A_{16} \end{array}$

11.

a) a = 1575, b = 231 r = a mod b = 189 Note: mod gives the remainder (r)

a = 231, b = 189 r = a mod b = 42
a = 189, b = 42 r = a mod b = 21
a =42, b = 21 r = a mod b =0 , gcd(1575,231) = 21

b) b = 3719, a = 8416 r = a mod b = 189
Note: gcd(8416, 3719) = gcd(-3719,8416)

a = 8416, b = 3719 r = a mod b = 978
a = 3719, b = 978 r = a mod b = 785
a =978, b = 785 r = a mod b =193 ,
a =785, b = 193 r = a mod b =13
a =193, b = 13 r = a mod b =11
a =13, b = 11 r = a mod b =2
a =13, b = 2 r = a mod b =1, gcd(8416, 3719) = 1

c) a = 28,844, b = 15,712 r = a mod b = 13132
Note: gcd(28844, 15712) = gcd(28844,-15712)

a = 15712, b = 13132 r = a mod b = 2580
a = 13132, b = 2580 r = a mod b = 232
a = 2580, b = 232 r = a mod b =28
a = 232, b = 28 r = a mod b =8
a = 28, b = 8 r = a mod b =4
a = 8, b = 4 r = a mod b =0
gcd(28844, 15712) = 4

12. lcm (a, b) = |a . b| / gcd(a, b)

a) lcm(1575,231) = | 1575 . 231| / 21 = | 363825/ 21| = 17325
b) lcm(8416,3719) = | 8416 . 3719| / 1 = | 363825/ 1| = 31299104

c) lcm(28844,15712) = | 28844 . 15712| / 4 = | 453196928/ 4| = 113299232

13. Prime numbers less than 200 are

    2 3 5 7 11 13 17 19 23 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97
    101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179
    181 191 193 197 199
    (see method developed in class (Power Point chapter 4).

14. Find a (mod n)

    a) a = 43,197, n = 39
       43197(mod 39) = 24
       43,197/ 39 = 1107.6 = 1107
       43,197 = 1107 (39) + 24

    b) a = -125, 617, n = 315
       -125,617(mod 315) = 68
       -125,617/ 315 = -398.7 = -399
       -125,617 = -399 (315) + 68

15. Find the solution of the following congruencies

    a) $x \equiv 7 \pmod 9$
       $9 \mid (x - 7)$, so $x = 7$ because $9 \mid 0$
    b) $x \equiv 4 \pmod{12}$
       $12 \mid (x - 4)$, so $x = 4$ because $12 \mid 0$
    c) $x \equiv 16 \pmod{21}$
       $21 \mid (x - 16)$, so $x = 16$ because $21 \mid 0$